

F-string Solution in $AdS_4 \times \mathcal{CP}^3$ PP-wave Background

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Abstract

We present supergravity solution for F-string in pp-wave background obtained from $AdS_4 \times \mathcal{CP}^3$ with zero flat directions. The classical solution is shown to break all space-time supersymmetries. We explicitly write down the standard as well as supernumerary Killing spinors both for the background and F-string solution.

Keywords: Supergravity, PP-wave background, $AdS_4 \times CP^3$, Supersymmetry, Killing spinors

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1 Introduction

Study of string theory in plane wave background [1–19] has drawn special attention, in the context of establishing AdS/CFT like dualities. In several cases, pp-waves are also known to be maximally supersymmetric solutions of supergravities in various dimensions [20–22]. Moreover, they are known to provide exactly solvable string theories from worldsheet point of view. In this connection, many classes of D-brane solutions are investigated in different pp-wave background geometry [23–27].

In this paper we are interested in a Penrose limit of the $AdS_4 \times \mathcal{CP}^3$ background considered in [28]. A good understanding of Type IIA string theory on $AdS_4 \times \mathcal{CP}^3$ is of great importance, due to its appearance in another example of AdS/CFT duality i.e. AdS_4/CFT_3 [29–31]. The new duality is motivated by the conjecture that $\mathcal{N} = 8$ Superconformal Chern-Simons theories in three dimensions describe dynamics of multiple M2-branes [32–46]. The ABJM model [47] describes a new example of the AdS/CFT duality involving $\mathcal{N} = 6$ superconformal $SU(N) \times SU(N)$ Chern-Simons theory in three dimensions and M-theory on $AdS_4 \times S^7/\mathbb{Z}_k$, where k is the level of the Chern-Simons action. At strong coupling, the M-theory on $AdS_4 \times S^7/\mathbb{Z}_k$, can be effectively described by type IIA superstring theory on the $AdS_4 \times \mathcal{CP}^3$ background [47]. Construction of string and brane actions for IIA strings in $AdS_4 \times \mathcal{CP}^3$ has been discussed extensively in [48–55]. In the aim to consider CFT's with a defect, $AdS/dCFT$ duality, both supersymmetric and non-supersymmetric embeddings of D -branes in $AdS_4 \times \mathcal{CP}^3$ have been described in [56]. In this context, pp-wave geometries arising from the $AdS_4 \times \mathcal{CP}^3$ has also been studied. Pp-wave backgrounds obtained by taking Penrose limit of $AdS_4 \times \mathcal{CP}^3$ with zero space like isometry [28], with one and two space like isometries [57] were studied. The case of a D2-brane in a general pp-wave background obtained from $AdS_4 \times \mathcal{CP}^3$ was considered in [58] with electric and magnetic fields turned on.

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In view of these developments, herewith we present the classical solution for fundamental string in pp-wave background obtained from $AdS_4 \times \mathcal{CP}^3$ with zero flat directions considered in [28]. We explicitly verify that our solutions satisfy the field equations. The supersymmetric properties of the solution is discussed by explicitly finding the standard as well as supernumerary Killing spinors. The rest of the paper is organised as follows. In section-2, we review the pp-wave backgrounds with different number of space like isometries. In section-3, we present the F-string solution in the $AdS_4 \times \mathcal{CP}^3$ pp-wave background. The supersymmetry analysis is discussed in section-4. We analyze the supersymmetry of the background and the F-string solution in this background by solving the Killing spinor equations. Section-5 contains our conclusions and discussions. In the appendix-6, we have listed the non-vanishing Christoffel symbols, Ricci tensors and Ricci scalar, those used for the calculation. For an example, we have presented the R_{++} equation of motion in detail in the appendix.

2 The pp-wave background of $AdS_4 \times \mathcal{CP}^3$

As a warm up exercise, in this section, we review few basic facts about the plane wave backgrounds obtained by taking Penrose limit of $AdS_4 \times \mathcal{CP}^3$ [28, 57, 58], which will be helpful in fixing the notations etc. Depending on the number of space-like isometries, we have three different pp-wave background arising from $AdS_4 \times \mathcal{CP}^3$. Starting from the general form of the metric, the three pp-wave backgrounds can be written down by choosing appropriate values for different parameters. The general form of pp-wave geometry is given by [57, 58],

$$ds^2 = -4dz^+dz^- + \sum_{i=1}^4 \left((du^i)^2 - (u^i)^2 (dz^+)^2 \right) + \sum_{j=1}^2 \left[(dz^j)^2 + (dy^j)^2 + \left(\xi_j^2 - \frac{1}{4} \right) ((z^j)^2 + (y^j)^2) (dz^+)^2 + 2 \left((\xi_j - 2C_j) z^j dy^j - (\xi_j + 2C_j) y^j dz^j \right) dz^+ \right] \quad (2.1)$$

and by choosing the C_j and ξ_j parameters appropriately, one arrives at the pp-wave background with no flat direction, one flat direction and two flat directions. In addition, there are two- and four-form RR fields.

It is easy to see that if we choose the C_j and ξ_j parameters to be zero, i.e.

$$\xi_j = C_j = 0 \quad (2.2)$$

the metric in eq(2.1) will be reduced to the following form,

$$ds^2 = -4dz^+dz^- - \left(\sum_{i=1}^4 (u^i)^2 + \frac{1}{4} \sum_{j=1}^2 ((z^j)^2 + (y^j)^2) \right) (dz^+)^2 + \sum_{i=1}^4 (du^i)^2 + \sum_{j=1}^2 (dz^j)^2 + \sum_{j=1}^2 (dy^j)^2 \quad (2.3)$$

This describes an $AdS_4 \times \mathcal{CP}^3$ pp-wave background with no explicit space-like isometry i.e. with no flat direction [28].

Again, for the following choices of the C_j and ξ_j parameters,

$$\xi_1 = \frac{1}{2}, \xi_2 = b + \frac{1}{2}, C_1 = \frac{1}{4}, C_2 = 0, \quad (2.4)$$

where b is an arbitrary parameter, one gets the metric [57] :

$$ds^2 = -4dz^+dz^- - \left(\sum_{i=1}^4 (u^i)^2 - b(b+1)((z^2)^2 + (y^2)^2) \right) (dz^+)^2 + (2b+1) [z^2 dy^2 - y^2 dz^2] dz^+ - 2y^1 dz^1 dz^+ + \sum_{i=1}^4 (du^i)^2 + \sum_{j=1}^2 (dz^j)^2 + \sum_{j=1}^2 (dy^j)^2$$

(2.5)

From eq(2.5), we find that z^1 is an explicit isometry of the background and hence this pp-wave background is referred as one flat direction pp-wave background.

If one chooses the C_j and ξ_j parameters to have the following values,

$$\xi_j = -\frac{1}{2}, C_j = \frac{1}{4} \quad (2.6)$$

then the metric given in eq(2.1) becomes

$$ds^2 = -4dz^+dz^- - \sum_{i=1}^4 (u^i)^2 (dz^+)^2 + \sum_{i=1}^4 (du^i)^2 + \sum_{j=1}^2 (dz^j)^2 + \sum_{j=1}^2 (dy^j)^2 - 2(z^1 dy^1 + z^2 dy^2) dz^+ \quad (2.7)$$

The metric in eq(2.7) describes exactly a pp-wave background with two flat directions [57], namely y^1 and y^2 .

In the present work, we shall be interested in the pp-wave background with zero flat directions. By considering the following coordinate transformation,

$$\begin{array}{ccccc} x^+ = z^+ & x^- = -2z^- & x^1 = u^1 & x^2 = u^2 & x^3 = u^3 \\ x^4 = u^4 & x^5 = z^1 & x^6 = y^1 & x^7 = z^2 & x^8 = y^2 \end{array} \quad (2.8)$$

we can rewrite the $AdS_4 \times \mathcal{CP}^3$ pp-wave background metric with zero flat directions given in eq(2.3) as:

$$ds^2 = 2dx^+dx^- - \left(\sum_{a=1}^4 (x^a)^2 + \frac{1}{4} \sum_{p=5}^8 (x^p)^2 \right) (dx^+)^2 + \sum_{a=1}^4 (dx^a)^2 + \sum_{p=5}^8 (dx^p)^2 \quad (2.9)$$

with the two- and four-form RR fields $F_{+4} = \frac{k}{2\tilde{R}}$ and $F_{+123} = \frac{3}{2} \frac{k}{\tilde{R}}$, where k is the level of the dual Chern-Simons gauge theory and \tilde{R} is the radius of AdS_4 and \mathcal{CP}^3 as defined in [28].¹The dilaton is expressed as $e^\phi = \frac{2\tilde{R}}{k}$. Defining $\mu = \frac{k}{2\tilde{R}}$, we can rewrite the values of RR-fluxes as $F_{+4} = \mu$, $F_{+123} = 3\mu$ and the dilaton as $e^\phi = \mu^{-1}$.

For the subsequent analysis of the present paper, we shall be interested in the following background.

$$ds^2 = 2dx^+dx^- - \left(\sum_{a=1}^4 x_a^2 + \frac{1}{4} \sum_{p=5}^8 x_p^2 \right) (dx^+)^2 + \sum_{a=1}^4 (dx^a)^2 + \sum_{p=5}^8 (dx^p)^2$$

with

$$F_{+4} = \mu, \quad F_{+123} = 3\mu. \quad (2.10)$$

The light-cone superstring action on the above pp-wave background (2.10) is studied in [59, 60] and found to be preserving 24 supercharges.

¹see ref. [28] for details.

3 Supergravity Solution

In this section, we present the classical solution corresponding to F-string on the above pp-wave background (2.10). We start by writing down the ansatz for the metric, the dilaton, the NS-NS B-field and RR-field strengths of such a configuration,

$$\begin{aligned}
ds^2 &= f^{-1} \left[2dx^+ dx^- - \left(\sum_{a=1}^4 (x^a)^2 + \frac{1}{4} \sum_{p=5}^8 (x^p)^2 \right) (dx^+)^2 \right] + \sum_{a=1}^4 (dx^a)^2 + \sum_{p=5}^8 (dx^p)^2 \\
e^{2\phi} &= \mu^{-2} f^{-1}, \\
(F_2)_{+4} &= \mu, \quad (F_4)_{+123} = 3\mu, \\
H_{+-a} &= \partial_a f^{-1}, \quad \forall a = 1, \dots, 4 \\
H_{+-p} &= \partial_p f^{-1}, \quad \forall p = 5, \dots, 8
\end{aligned} \tag{3.1}$$

where f is a harmonic function in the 8-dimensional transverse space. We have explicitly verified that the above solution satisfies the type IIA field equations listed in [61]. In particular, we found that the R_{++} equation of motion will be satisfied if f satisfies the following condition,²

$$\sum_{a=1}^4 \frac{\partial^2 f}{\partial x^{a2}} + \sum_{p=5}^8 \frac{\partial^2 f}{\partial x^{p2}} = 0 \tag{3.2}$$

4 Supersymmetry analysis

In this section, we analyze the supersymmetry of the solution presented above in Section-3. We start by writing down the supersymmetry variations of dilatino and gravitino fields of type IIA supergravity in ten dimensions, in string frame, [62] :

$$\begin{aligned}
\delta\lambda_{\pm} &= \frac{1}{2}(\Gamma^{\mu}\partial_{\mu}\phi \mp \frac{1}{12}\Gamma^{\mu\nu\rho}H_{\mu\nu\rho})\epsilon_{\pm} + \\
&\quad \frac{1}{8}e^{\phi}(5F^0 \pm \frac{3}{2!}\Gamma^{\mu\nu}F_{\mu\nu}^{(2)} + \frac{1}{4!}\Gamma^{\mu\nu\rho\alpha}F_{\mu\nu\rho\alpha}^{(4)})\epsilon_{\mp},
\end{aligned} \tag{4.1}$$

$$\begin{aligned}
\delta\Psi_{\mu}^{\pm} &= \left[\partial_{\mu} + \frac{1}{4}(w_{\mu\hat{\nu}\hat{\rho}} \mp \frac{1}{2}H_{\mu\hat{\nu}\hat{\rho}})\Gamma^{\hat{\nu}\hat{\rho}} \right]\epsilon_{\pm} \\
&+ \frac{1}{8}e^{\phi} \left[F^0 \pm \frac{1}{2!}\Gamma^{\mu\nu}F_{\mu\nu}^{(2)} + \frac{1}{4!}\Gamma^{\mu\nu\rho\alpha}F_{\mu\nu\rho\alpha}^{(4)} \right]\Gamma_{\mu}\epsilon_{\mp},
\end{aligned} \tag{4.2}$$

where we have used (μ, ν, ρ) to describe the ten dimensional space-time indices, and hat's represent the corresponding tangent space indices.

4.1 Background Supersymmetry

Before analyzing the supersymmetry of the F-string solution in the plane wave background of $AdS_4 \times \mathcal{CP}^3$ space-time, let's first discuss the supersymmetry of the background itself. The dilatino (4.1) and gravitino (4.2) variations impose non-trivial conditions on the spinor ϵ_{\pm} .

Using the indices: $(+, -, a, p)$ to denote the ten dimensional coordinates with $a = 1, \dots, 4$ and $p = 5, \dots, 8$, we get the following condition from the dilatino equation for the $AdS_4 \times \mathcal{CP}^3$ pp-wave background given in eq(2.10)³ (hats denoting the corresponding tangent space coordinates):

$$\left[\Gamma^{\hat{+}\hat{4}} \mp \Gamma^{\hat{+}\hat{1}\hat{2}\hat{3}} \right] \epsilon_{\pm} = 0 \tag{4.3}$$

²see Appendix for details

³Note that $F^0 = 0$ for the pp-wave background eq(2.10) under consideration.

Gravitino variation gives the following conditions on the spinors: ⁴

$$\delta\psi_{+}^{\pm} = \partial_{+}\epsilon_{\pm} - \frac{1}{2}[x_{\hat{a}}\Gamma^{\hat{a}} + \frac{1}{4}x_{\hat{p}}\Gamma^{\hat{p}}]\epsilon_{\pm} + \frac{1}{8}[\pm\Gamma^{\hat{4}} + 3\Gamma^{\hat{1}\hat{2}\hat{3}}]\Gamma^{\hat{c}}\epsilon_{\mp} = 0 \quad (4.4)$$

$$\delta\psi_{-}^{\pm} = \partial_{-}\epsilon_{\pm} = 0 \quad (4.5)$$

$$\delta\psi_a^{\pm} = \partial_a\epsilon_{\pm} + \frac{1}{8}[\pm\Gamma^{\hat{4}} + 3\Gamma^{\hat{1}\hat{2}\hat{3}}]\delta_{a\hat{a}}\Gamma^{\hat{a}}\epsilon_{\mp} = 0, \quad \forall a = 1, \dots, 4 \quad (4.6)$$

$$\delta\psi_p^{\pm} = \partial_p\epsilon_{\pm} + \frac{1}{8}[\pm\Gamma^{\hat{4}} + 3\Gamma^{\hat{1}\hat{2}\hat{3}}]\delta_{p\hat{p}}\Gamma^{\hat{p}}\epsilon_{\mp} = 0, \quad \forall p = 5, \dots, 8 \quad (4.7)$$

Now there are two kinds of solutions of the above equations eq(4.3)- eq(4.7). One corresponds to considering $\Gamma^{\hat{c}}\epsilon_{\pm} = 0$, those are called ‘standard’ or ‘normal’ Killing spinors [27, 63, 64]. This condition keeps 16 spinors out of the set of total 32. Rest of the killing spinors for which $\Gamma^{\hat{c}}\epsilon_{\pm} \neq 0$, are usually known as ‘supernumerary’ killing spinors.

Let’s first consider the ‘normal’ killing spinors with the condition $\Gamma^{\hat{c}}\epsilon_{\pm} = 0$. For these spinors except eq(4.4) all other equations can be trivially satisfied. The eq(4.4)- eq(4.7) are replaced by:

$$\partial_{+}\epsilon_{\pm} - \frac{1}{4}[\pm\Gamma^{\hat{4}} + 3\Gamma^{\hat{1}\hat{2}\hat{3}}]\epsilon_{\mp} = 0 \quad (4.8)$$

$$\partial_{-}\epsilon_{\pm} = 0, \quad \partial_a\epsilon_{\pm} = 0, \quad \partial_p\epsilon_{\pm} = 0 \quad (4.9)$$

Eq(4.9) implies that the spinors ϵ_{\pm} are independent of x^{-} , x^a and x^p . Eq(4.8) can now be solved easily after imposing either of the following two projections:

$$\Gamma^{\hat{1}\hat{2}\hat{3}\hat{4}}\epsilon_{\pm} = \pm\epsilon_{\pm} \quad (4.10)$$

$$\Gamma^{\hat{1}\hat{2}\hat{3}\hat{4}}\epsilon_{\pm} = \mp\epsilon_{\pm} \quad (4.11)$$

Explicitly, considering the projection eq(4.10), eq(4.8) reduces to:

$$\partial_{+}\epsilon_{\pm} - \Gamma^{\hat{1}\hat{2}\hat{3}}\epsilon_{\mp} = 0 \quad (4.12)$$

The Killing spinor equations are then solved by spinors: $\eta_{\pm} = \epsilon_{+} \pm \epsilon_{-} = \exp[\pm\Gamma^{\hat{1}\hat{2}\hat{3}}x^{+}]\eta_0$, with η_0 being a constant spinor. Similarly one can easily check that there are another set of spinors coming from the projection eq(4.11).

Next we consider the ‘supernumerary’ killing spinors with the condition $\Gamma^{\hat{c}}\epsilon_{\pm} \neq 0$. These spinors will be constrained by the condition:

$$[\pm\Gamma^{\hat{4}} + \Gamma^{\hat{1}\hat{2}\hat{3}}]\epsilon_{\mp} = 0 \quad (4.13)$$

We now use eq(4.13) to simplify the spinor equations eq(4.4) - eq(4.7) further. The dilatino variation condition eq(4.3) is now trivially satisfied and eqs(4.4)- (4.7), following from $\delta\Psi^{\pm}_{\mu} = 0$ can be written as:

$$\partial_{+}\epsilon_{\pm} - \frac{1}{2}[x_{\hat{a}}\Gamma^{\hat{a}} + \frac{1}{4}x_{\hat{p}}\Gamma^{\hat{p}}]\epsilon_{\pm} + \frac{1}{4}\Gamma^{\hat{1}\hat{2}\hat{3}}\Gamma^{\hat{c}}\epsilon_{\mp} = 0 \quad (4.14)$$

$$\partial_{-}\epsilon_{\pm} = 0 \quad (4.15)$$

$$\partial_a\epsilon_{\pm} + \frac{1}{2}\Gamma^{\hat{1}\hat{2}\hat{3}}\delta_{a\hat{a}}\Gamma^{\hat{a}}\epsilon_{\mp} = 0, \quad \forall a = 1, \dots, 4 \quad (4.16)$$

$$\partial_p\epsilon_{\pm} + \frac{1}{4}\Gamma^{\hat{1}\hat{2}\hat{3}}\delta_{p\hat{p}}\Gamma^{\hat{p}}\epsilon_{\mp} = 0, \quad \forall p = 5, \dots, 8 \quad (4.17)$$

⁴We are in the frame where $(\Gamma^{\hat{+}})^2 = (\Gamma^{\hat{-}})^2 = 0$, $[\Gamma^{\hat{+}}, \Gamma^{\hat{-}}]_{-} = 2$. We have made use of the identities: $\Gamma^{\hat{+}}\Gamma^{\hat{+}\hat{-}} = -\Gamma^{\hat{+}}$, $\Gamma^{\hat{+}\hat{-}}\Gamma^{\hat{-}} = -\Gamma^{\hat{-}}$ etc..

The above equations eq(4.14) - eq(4.17) can be solved for spinors $\eta_{\pm} = \epsilon_{+} \pm \epsilon_{-}$. Adding the upper and lower sign equations in eq(4.14) - eq(4.17) lead to the differential equations for η_{+} , while subtracting the two give those for η_{-} . The gravitini variations in terms of η_{+} can be written as:

$$\partial_{+}\eta_{+} - \frac{1}{2}[x_{\hat{a}}\Gamma^{\hat{a}} + \frac{1}{4}x_{\hat{p}}\Gamma^{\hat{p}}]\eta_{+} + \frac{1}{4}\Gamma^{\hat{1}\hat{2}\hat{3}}\Gamma^{\hat{+}}\eta_{+} = 0 \quad (4.18)$$

$$\partial_a\eta_{+} + \frac{1}{2}\Gamma^{\hat{1}\hat{2}\hat{3}}\delta_{a\hat{a}}\Gamma^{\hat{a}}\eta_{+} = 0 \quad (4.19)$$

$$\partial_p\eta_{+} + \frac{1}{4}\Gamma^{\hat{1}\hat{2}\hat{3}}\delta_{p\hat{p}}\Gamma^{\hat{p}}\eta_{+} = 0 \quad (4.20)$$

The x^{-} component of the Killing spinor equation eq(4.15) is found to be reduced to $\partial_{-}\eta_{+} = 0$, which implies that η_{+} is independent of x^{-} . The x^a and x^p components of the Killing spinor equation can be written as $(\partial_a + \Omega_a)\eta_{+} = 0$ and $(\partial_p + \Omega_p)\eta_{+} = 0$ respectively. Since $\Omega_a\Omega_b = 0 = \Omega_p\Omega_q$ for any $a, b = 1 \cdots 4$ and any $p, q = 5 \cdots 8$ using the condition $(\Gamma^{\hat{+}})^2 = 0$, the x^a as well as x^p dependence of η_{+} is expected to be linear. The solution of the above Killing spinor equations eq(4.18) - eq(4.20) is then found in a similar way as in [20, 27, 63] and is given as:

$$\eta_{+} = (1 - \Omega_a x^a - \Omega_p x^p) \exp \left[-\frac{1}{4}\Gamma^{\hat{1}\hat{2}\hat{3}}\Gamma^{\hat{+}}x^{+} \right] \cdot \eta_0 \quad (4.21)$$

with η_0 being a constant spinor and Ω_a 's, Ω_p 's are given by,

$$\Omega_a = \frac{1}{2}\Gamma^{\hat{+}}\Gamma^{\hat{1}\hat{2}\hat{3}}\delta_{a\hat{a}}\Gamma^{\hat{a}}, \quad \Omega_p = \frac{1}{4}\Gamma^{\hat{+}}\Gamma^{\hat{1}\hat{2}\hat{3}}\delta_{p\hat{p}}\Gamma^{\hat{p}} \quad (4.22)$$

One can easily write down the gravitini variations in terms of η_{-} and verify that the spinor equations are satisfied with spinors: $\eta_{-} = (1 + \Omega_a x^a + \Omega_p x^p) \exp \left[\frac{1}{4}\Gamma^{\hat{1}\hat{2}\hat{3}}\Gamma^{\hat{+}}x^{+} \right] \cdot \eta_0$.

The 16 standard Killing spinors and 8 supernumerary ones give a total of 24 Killing spinors for the type IIA pp-wave background presented in eq(2.10). Thus the total number of supersymmetries preserved by the background pp-wave will be $\frac{3}{4}$ of the maximal ones. We thus confirm the results discussed in [59, 60].

4.2 F-string supersymmetry

In this section we will present the supersymmetry of the F-string solution described earlier in section-3. We will see below that in spite of its simple form, this solution does not preserve any space-time supersymmetry. Solving the supersymmetry variations of dilatino and gravitino fields given in eq(4.1) - eq(4.2) for the solution describing an F-string in eq(3.1) ⁵, we get several conditions. The vanishing of the dilatino variations give

$$\delta\lambda_{\pm} = \frac{f_{,\hat{a}}}{4f} \left[-\Gamma^{\hat{a}} \pm \Gamma^{\hat{+}\hat{-}\hat{a}} \right] \epsilon_{\pm} + \frac{f_{,\hat{p}}}{4f} \left[-\Gamma^{\hat{p}} \pm \Gamma^{\hat{+}\hat{-}\hat{p}} \right] \epsilon_{\pm} + \frac{3}{8} \left[\pm \Gamma^{\hat{+}\hat{4}} + \Gamma^{\hat{+}\hat{1}\hat{2}\hat{3}} \right] \epsilon_{\mp} = 0. \quad (4.23)$$

Eq(4.23) to be hold, we should have the following conditions satisfied:

$$\left[\Gamma^{\hat{a}} \mp \Gamma^{\hat{+}\hat{-}\hat{a}} \right] \epsilon_{\pm} = 0, \quad \left[\Gamma^{\hat{p}} \mp \Gamma^{\hat{+}\hat{-}\hat{p}} \right] \epsilon_{\pm} = 0 \quad (4.24)$$

$$\left[\Gamma^{\hat{+}\hat{4}} \mp \Gamma^{\hat{+}\hat{1}\hat{2}\hat{3}} \right] \epsilon_{\pm} = 0 \quad (4.25)$$

⁵Here also we have $F^0 = 0$

Gravitino variation gives the following conditions on the spinors:

$$\delta\psi_+^\pm = \partial_+\epsilon_\pm - \frac{1}{2f^{1/2}}[x_{\hat{a}}\Gamma^{\hat{a}} + \frac{1}{4}x_{\hat{p}}\Gamma^{\hat{p}}]\epsilon_\pm + \frac{1}{8f^{1/2}}[\pm\Gamma^{\hat{4}} + 3\Gamma^{\hat{1}\hat{2}\hat{3}}]\Gamma^{\hat{c}}\epsilon_\mp = 0 \quad (4.26)$$

$$\delta\psi_-^\pm = \partial_-\epsilon_\pm = 0 \quad (4.27)$$

$$\delta\psi_a^\pm = \partial_a\epsilon_\pm + \frac{1}{4}\frac{f_{,a}}{f}\epsilon_\pm + \frac{1}{8}[\pm\Gamma^{\hat{4}} + 3\Gamma^{\hat{1}\hat{2}\hat{3}}]\delta_{a\hat{a}}\Gamma^{\hat{a}}\epsilon_\mp = 0, \quad \forall a = 1, \dots, 4 \quad (4.28)$$

$$\delta\psi_p^\pm = \partial_p\epsilon_\pm + \frac{1}{4}\frac{f_{,p}}{f}\epsilon_\pm + \frac{1}{8}[\pm\Gamma^{\hat{4}} + 3\Gamma^{\hat{1}\hat{2}\hat{3}}]\delta_{p\hat{p}}\Gamma^{\hat{p}}\epsilon_\mp = 0, \quad \forall p = 5, \dots, 8 \quad (4.29)$$

In writing the above set of conditions, we have used eq(4.24).

First, looking for the ‘normal’ Killing spinors satisfying the condition $\Gamma^{\hat{4}}\epsilon_\pm = 0$. In addition, we have to use either of the two projections given in eq(4.10) and eq(4.11). Proceeding with the projection condition eq(4.10), the gravitini equations from eq(4.26) - eq(4.29) to be solved are:

$$\partial_+\epsilon_\pm - \frac{1}{f^{1/2}}\Gamma^{\hat{1}\hat{2}\hat{3}}\epsilon_\mp = 0, \quad (4.30)$$

$$\partial_a\epsilon_\pm + \frac{1}{4}\frac{f_{,a}}{f}\epsilon_\pm = 0, \quad \forall a = 1, \dots, 4 \quad (4.31)$$

$$\partial_p\epsilon_\pm + \frac{1}{4}\frac{f_{,p}}{f}\epsilon_\pm = 0, \quad \forall p = 5, \dots, 8 \quad (4.32)$$

Adding the upper and lower sign equations in eq(4.30) - eq(4.32), we have the killing spinor equations for $\eta_+ = \epsilon_+ + \epsilon_-$ and subtracting them we get the corresponding ones for $\eta_- = \epsilon_+ - \epsilon_-$. Let us first consider the gravitini variations for η_+ :

$$\partial_+\eta_+ - \frac{1}{f^{1/2}}\Gamma^{\hat{1}\hat{2}\hat{3}}\eta_+ = 0, \quad (4.33)$$

$$\partial_a\eta_+ + \frac{1}{4}\frac{f_{,a}}{f}\eta_+ = 0 \quad (4.34)$$

$$\partial_p\eta_+ + \frac{1}{4}\frac{f_{,p}}{f}\eta_+ = 0 \quad (4.35)$$

One can check that eq(4.33) - eq(4.35) are non integrable ones due to the specific form of the function f . We have explicitly checked that the equations for η_- are also non integrable.

Next, we will analyze the ‘supernumerary’ Killing spinors. The vanishing of the dilatino variation implies that these spinors will be constrained by eq(4.13) in addition to eq(4.24). The gravitini variations eq(4.26) - eq(4.29) can be simplified further using eq(4.13) and eq(4.24) and can be written as :

$$\partial_+\epsilon_\pm - \frac{1}{2f^{1/2}}[x_{\hat{a}}\Gamma^{\hat{a}} + \frac{1}{4}x_{\hat{p}}\Gamma^{\hat{p}}]\epsilon_\pm + \frac{1}{4f^{1/2}}\Gamma^{\hat{1}\hat{2}\hat{3}}\Gamma^{\hat{c}}\epsilon_\mp = 0 \quad (4.36)$$

$$\partial_-\epsilon_\pm = 0 \quad (4.37)$$

$$\partial_a\epsilon_\pm + \frac{1}{4}\frac{f_{,a}}{f}\epsilon_\pm + \frac{1}{2}\Gamma^{\hat{1}\hat{2}\hat{3}}\delta_{a\hat{a}}\Gamma^{\hat{a}}\epsilon_\mp = 0, \quad \forall a = 1, \dots, 4 \quad (4.38)$$

$$\partial_p\epsilon_\pm + \frac{1}{4}\frac{f_{,p}}{f}\epsilon_\pm + \frac{1}{4}\Gamma^{\hat{1}\hat{2}\hat{3}}\delta_{p\hat{p}}\Gamma^{\hat{p}}\epsilon_\mp = 0, \quad \forall p = 5, \dots, 8 \quad (4.39)$$

Now, one can easily write down the equations in the basis $\eta_\pm = \epsilon_+ \pm \epsilon_-$ and check that these are non integrable ones due to specific dependence of f . This shows that the F-string solution in equation (3.1) does not preserve any of the space-time supersymmetries.

5 Summary and Discussion

In this paper we have found the supergravity solution of fundamental string in the pp-wave background arising from the Penrose limit of $AdS_4 \times \mathcal{CP}^3$ with zero space like isometries presented in [28]. We discussed the supersymmetry of the pp-wave background and also the F-string solution by solving the type-II Killing spinor equations explicitly. We confirmed that the background preserves 24 Killing spinors out of which 16 are of 'standard' type and rest 8 are 'supernumerary' in nature. It is shown that the classical F-string solution on this background does not preserve any space-time supersymmetry. The corresponding worldsheet analysis may reveal the reason for the solution destroying all space-time supersymmetry. There is a possibility that while supersymmetry is preserved on the worldsheet it does not admit a local space-time realization.⁶ One could possibly try to study the stability of the background. It is interesting to investigate the F-string solution in the pp-wave backgrounds with one flat direction and two flat directions. It might also be possible to use series of T- and S- duality to generate other solutions for Dp-branes in this background. We hope to return to these questions in our future exercise. The solution obtained in this paper seems very interesting in the line of supergravity theories. However, the physical significance of the solution remain elusive from us as of now.

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6 Appendix

In this appendix, we summarize the nonvanishing christoffel symbols, Ricci tensors and Ricci scalar obtained from the eq(3.1):

$$\Gamma_{+a}^+ = -\frac{1}{2f} \frac{\partial f}{\partial x^a}, \quad \Gamma_{+p}^+ = -\frac{1}{2f} \frac{\partial f}{\partial x^p}, \quad \Gamma_{+a}^- = -x^a, \quad (6.1)$$

$$\Gamma_{++}^a = \frac{x^a}{f} - \frac{1}{2f^2} \frac{\partial f}{\partial x^a} \left[\sum_{a=1}^4 (x^a)^2 + \frac{1}{4} \sum_{p=5}^8 (x^p)^2 \right], \quad \Gamma_{-a}^- = -\frac{1}{2f} \frac{\partial f}{\partial x^a}, \quad \Gamma_{-p}^- = -\frac{1}{2f} \frac{\partial f}{\partial x^p} \quad (6.2)$$

$$\Gamma_{++}^p = \frac{x^p}{4f} - \frac{1}{2f^2} \frac{\partial f}{\partial x^p} \left[\sum_{a=1}^4 (x^a)^2 + \frac{1}{4} \sum_{p=5}^8 (x^p)^2 \right] \quad \Gamma_{+-}^a = \frac{1}{2} \frac{1}{f^2} \frac{\partial f}{\partial x^a}, \quad \Gamma_{+-}^p = \frac{1}{2} \frac{1}{f^2} \frac{\partial f}{\partial x^p} \quad (6.3)$$

$$\Gamma_{+p}^- = -\frac{x^p}{4} \quad (6.4)$$

And the Ricci tensors thus obtained are given by:

$$\begin{aligned} R_{++} = & \frac{1}{f^3} \left(\sum_{a=1}^4 x^{a2} + \sum_{p=5}^8 \frac{x^{p2}}{4} \right) \left[\sum_{a=1}^4 \left(\frac{\partial f}{\partial x^a} \right)^2 + \sum_{p=5}^8 \left(\frac{\partial f}{\partial x^p} \right)^2 \right] \\ & - \frac{1}{2f^2} \left(\sum_{a=1}^4 x^{a2} + \sum_{p=5}^8 \frac{x^{p2}}{4} \right) \left[\sum_{a=1}^4 \frac{\partial^2 f}{\partial x^{a2}} + \sum_{p=5}^8 \frac{\partial^2 f}{\partial x^{p2}} \right] \\ & - \frac{1}{f^2} \left[\sum_{a=1}^4 x^a \left(\frac{\partial f}{\partial x^a} \right) + \sum_{p=5}^8 \frac{x^p}{4} \left(\frac{\partial f}{\partial x^p} \right) \right] + \frac{5}{f} \end{aligned} \quad (6.5)$$

⁶ We thank the unknown reviewer for the useful discussions on this issue.

$$R_{+-} = \frac{1}{2f^2} \left[\sum_{a=1}^4 \frac{\partial^2 f}{\partial x^{a2}} + \sum_{p=5}^8 \frac{\partial^2 f}{\partial x^{p2}} \right] - \frac{1}{f^3} \left[\sum_{a=1}^4 \left(\frac{\partial f}{\partial x^a} \right)^2 + \sum_{p=5}^8 \left(\frac{\partial f}{\partial x^p} \right)^2 \right] \quad (6.6)$$

$$R_{ab} = \frac{1}{f} \left[\sum_{a,b=1}^4 \frac{\partial}{\partial x^b} \left(\frac{\partial f}{\partial x^a} \right) \right] - \frac{3}{2f^2} \left[\sum_{a,b=1}^4 \left(\frac{\partial f}{\partial x^a} \right) \left(\frac{\partial f}{\partial x^b} \right) \right] \quad (6.7)$$

$$R_{pq} = \frac{1}{f} \left[\sum_{p,q=5}^8 \frac{\partial}{\partial x^q} \left(\frac{\partial f}{\partial x^p} \right) \right] - \frac{3}{2f^2} \left[\sum_{p,q=5}^8 \left(\frac{\partial f}{\partial x^p} \right) \left(\frac{\partial f}{\partial x^q} \right) \right] \quad (6.8)$$

$$R_{ap} = -\frac{3}{2f^2} \left[\sum_{a=1}^4 \frac{\partial f}{\partial x^a} \sum_{p=5}^8 \frac{\partial f}{\partial x^p} \right] + \frac{1}{f} \left[\sum_{a=1}^4 \sum_{p=5}^8 \frac{\partial}{\partial x^p} \left(\frac{\partial f}{\partial x^a} \right) \right] \quad (6.9)$$

The Ricci scalar is given as:

$$R = -\frac{7}{2f^2} \left[\sum_{a=1}^4 \left(\frac{\partial f}{\partial x^a} \right)^2 + \sum_{p=5}^8 \left(\frac{\partial f}{\partial x^p} \right)^2 \right] + \frac{2}{f} \left[\sum_{a=1}^4 \left(\frac{\partial^2 f}{\partial x^{a2}} \right) + \sum_{p=5}^8 \left(\frac{\partial^2 f}{\partial x^{p2}} \right) \right] \quad (6.10)$$

The background with the ansatz for the metric, the dilaton, the NS-NS B-field and RR field strengths, described in eq(3.1) satisfies the type IIA field equations listed in [61]. For an example let's consider the R_{++} equation of motion explicitly. Solving eq(67) of [61] for $i, j = +$ with the values for the dilaton, the NS-NS B-field and RR field strengths as given in eq(3.1), one gets:

$$\begin{aligned} R_{++} = & \frac{1}{f^3} \left(\sum_{a=1}^4 x^{a2} + \sum_{p=5}^8 \frac{x^{p2}}{4} \right) \left[\sum_{a=1}^4 \left(\frac{\partial f}{\partial x^a} \right)^2 + \sum_{p=5}^8 \left(\frac{\partial f}{\partial x^p} \right)^2 \right] \\ & - \frac{1}{f^2} \left[\sum_{a=1}^4 x^a \left(\frac{\partial f}{\partial x^a} \right) + \sum_{p=5}^8 \frac{x^p}{4} \left(\frac{\partial f}{\partial x^p} \right) \right] + \frac{5}{f} \end{aligned} \quad (6.11)$$

Clearly, eq(6.5) is in exact match with eq(6.11) when f satisfies eq(3.2).

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